Advanced International Journal of Multidisciplinary Research
E-ISSN: 2584-0487
editor@aijmr.com
Volume 4, Issue 3, May-June 2026
CrossRef DOI: 10.62127/aijmr.2026.v04i03.1305

Vertex Antimagic Edge Slither Labeling of Path Related Graphs

R. Sreenivasan

Assistant Professor
PG & Research Department of Mathematics
Agurchand Manmull Jain College, Meenambakkam, Chennai, India

Abstract:
The path graphs are one of the most intriguing topics in graph theory having a gamut of applications that extends to all spheres of science and technology. From daily life to sophisticated technology, the path graphs have imprinted their influence flawlessly. In this paper, the path related graphs, the comb graph and the double arrow graphs are considered and applicability of the vertex antimagic edge slither labeling is ensured.

Keywords: Antimagic Labeling, Antimagic Slither Labeling, Path Graph, Comb Graph, Arrow Graph, Double Arrow Graph and Umbrella Graph

Introduction:
The term labeling in graphs denotes the idea of allocating positive integers to the vertices or the edges or the both. In the modern days, the idea of labeling has been used by the research scholar's worldwide who have come out with different types of labeling for different graph structures. The scholars have not only applied labeling to just graphs, but they have represented many complex structures and situations using labeling. In graph theoretic parlance the notion of a path has been used effectively to elucidate a variety of concepts. In this paper, different manipulations of path graphs, the comb graph, arrow graph, double arrow graph and umbrella graphs are considered.

Basic Terminologies:
Antimagic Labeling: Martin Baca and Mirka Miller [5] were the first scholars to give the idea of antimagic magic labeling. R. Bodendiek and G. Walther [1] were the first to define the concept of (a, d) antimagic labeling and the definition is given as follows; "A graph G is said to accept (a, d) vertex antimagic edge labeling if there exists a positive integer a and a non-negative integer d and a bijective map given by the function $g_1{:}E\rightarrow\{1,2,\ldots,|E(G)|\}$ such that the induced mapping $g_2{:}V(G)\rightarrow W$, where $W = \{a,a+d,a+2d,\ldots,a+|V-1|d\}$ is also a bijection".
Slither Labeling: The idea of slither labeling arose from the idea of antimagic labeling where the labeling of edges is used in the form of a slither pattern. The labeling is done in such a way that the labels of the edges are never same. The label of a vertex is the sum of labels of the edges that are incident with it.
Path: A path is an alternate sequence of vertices and edges such that the vertices and edges of the graph are non-repeating.
Comb Graph: The comb graph CP_n considered here is due to P. Mythili and S. Gokilamani [7] is a graph that is formed from a path P_n in such a way that there are pendant vertices attached to all the vertices in the path. The number of vertices in the graph is 2n and the number of edges in the graph is 2n-1.

E-ISSN: 2584-0487
editor@aijmr.com
Volume 4, Issue 3, May-June 2026
CrossRef DOI: 10.62127/aijmr.2026.v04i03.1305

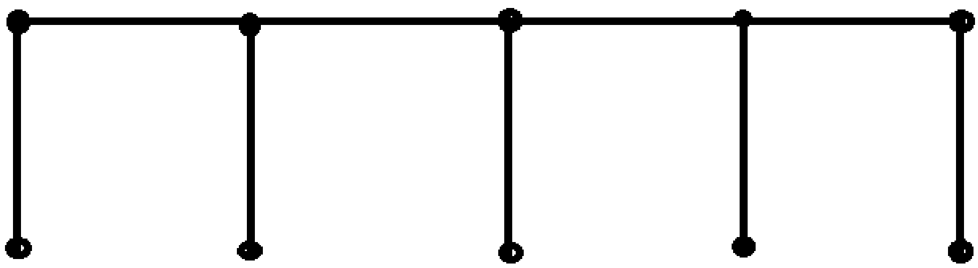

Fig 1. Comb graph CP_5.

Arrow Graph: An arrow graph AL_n is obtained from a ladder graph by attaching a triangle to the top end of the ladder graph. The number of vertices in the graph is 2n + 1 and the number of edges is 3n.

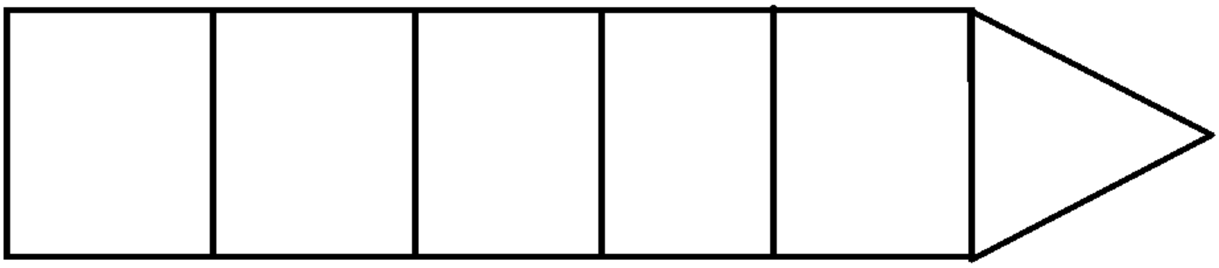

Fig 2. Arrow Graph AL_6.

Double Arrow Graph: An arrow graph DAL_n is obtained from a ladder graph by attaching a triangle to the top and the bottom end of the ladder graph. The number of number of vertices is 2n + 2 and the number of edges is 3n + 2.

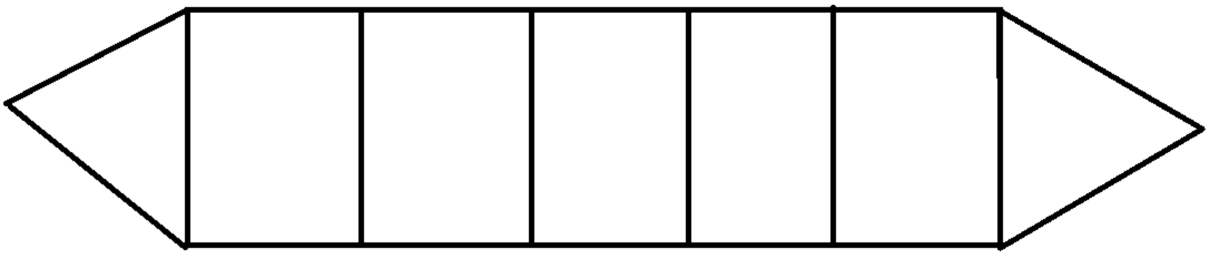

Fig 3. Arrow Graph DAL_6.

Umbrella Graph: An umbrella $U_{m,n}$ graph is constructed by connecting the path graph P_n to a central vertex of the fan graph f_m. The number of vertices and edges in the graph is m + n and 2m + n – 2 respectively. The umbrella graph that is under consideration is due to Lavanya. S and Ganesan. V [4].

Advanced International Journal of Multidisciplinary Research

E-ISSN: 2584-0487
editor@aijmr.com

Volume 4, Issue 3, May-June 2026

CrossRef DOI: 10.62127/aijmr.2026.v04i03.1305

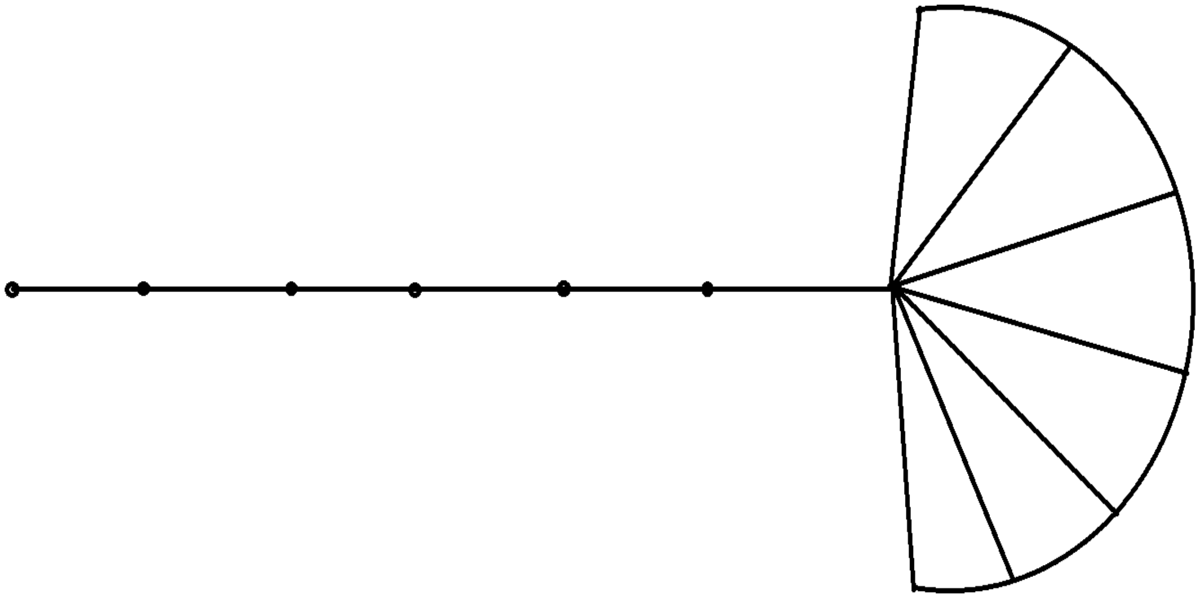

Fig 4. Umbrella Graph $U_{7,7}$.

Main Result:

Theorem – 1: The Comb graph CP_n admits vertex antimagic edge slither labeling for all $n \geq 2$.

Proof: Let CP_n be the comb graph formed on a path on n vertices. Label the edges of the graph in the slither formation using positive integers such that the labels of the integers are all different. The label of a vertex is the sum of labels of the edges that are incident with it. Let n = 7 be an arbitrary number such that the comb graph is formed on a path with 6 vertices. The edge slither labeling in the graph is given as follows;

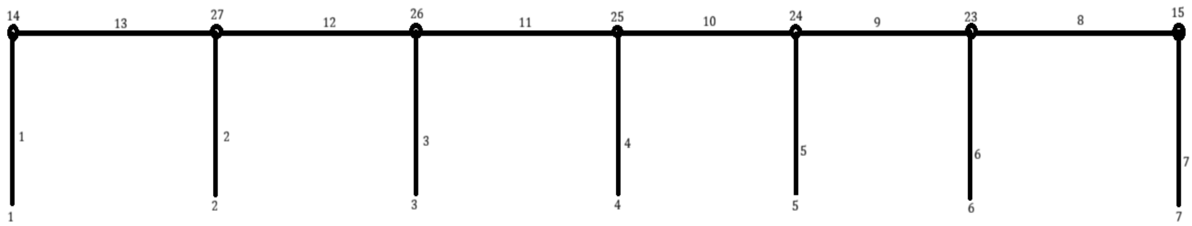

Fig 5. Vertex Antimagic Edge Slither Labeling in CP_7.

In the above graph, the edge labels are in slither pattern and the vertex labels are all distinct and hence the graph is vertex antimagic edge slither labelled. Since n = 7 is arbitrary, it follows that the comb graph CP_n admits vertex antimagic edge slither labeling for all $n \geq 2$.

Theorem – 2: The arrow graph AL_n admits vertex antimagic edge slither labeling for all $n \geq 2$.

Proof: Consider the arrow graph AL_n that is formed on L_n ladder. Label the edges of the graph in the slither pattern in such a way that the edge labels are all different. The label of a vertex is the sum of labels of the edges that are incident with it. Let n = 7 be an arbitrary number such that the ladder graph has 7 steps with a triangle mounted on the top of the ladder graph.

Advanced International Journal of Multidisciplinary Research

E-ISSN: 2584-0487
editor@aijmr.com

Volume 4, Issue 3, May-June 2026

CrossRef DOI: 10.62127/aijmr.2026.v04i03.1305

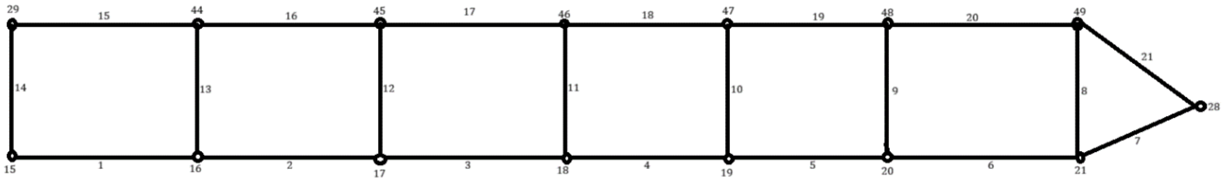

Fig 6. Vertex Antimagic Edge Slither Labeling in AL_7.

In the graph taken above, the edge labels are in slither pattern and are all distinct and so are the vertex labels and hence the graph is vertex antimagic edge slither labelled. Since n = 7 is arbitrary, it follows that the arrow graph AL_n admits vertex antimagic edge slither labeling for all $n \geq 2$.

Theorem – 3: The double arrow graph DAL_n admits vertex antimagic edge slither labeling.
Proof: Consider the double arrow graph DAL_n that is formed on L_n ladder. Label the edges of the graph in the slither pattern in such a way that the edge labels are all different. The label of a vertex is the sum of labels of the edges that are incident with it. Let n = 7 be an arbitrary number such that the ladder graph has 7 steps with a triangle mounted on the top and bottom of the ladder graph.

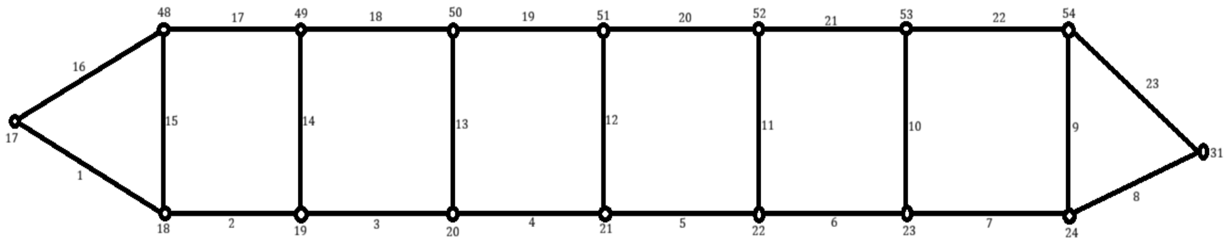

Fig 7. Vertex Antimagic Edge Slither Labeling in DAL_7.

In the graph taken above, the edge labels are in slither pattern and are all distinct and so are the vertex labels and hence the graph is vertex antimagic edge slither labelled. Since n = 7 is arbitrary, it follows that the double arrow graph DAL_n admits vertex antimagic edge slither labeling for all n.

Theorem – 4: The Umbrella graph $U_{m,n}$ admits vertex antimagic edge slither labeling.
Proof: Consider the umbrella graph $U_{m,n}$ that is formed on P_n path graph and F_m fan graph. Label the edges of the graph in the slither pattern in such a way that the edge labels are all different. The label of a vertex is the sum of labels of the edges that are incident with it. Let m = n = 7 be an arbitrary number such that the path and the fan graph constitute the umbrella graph.

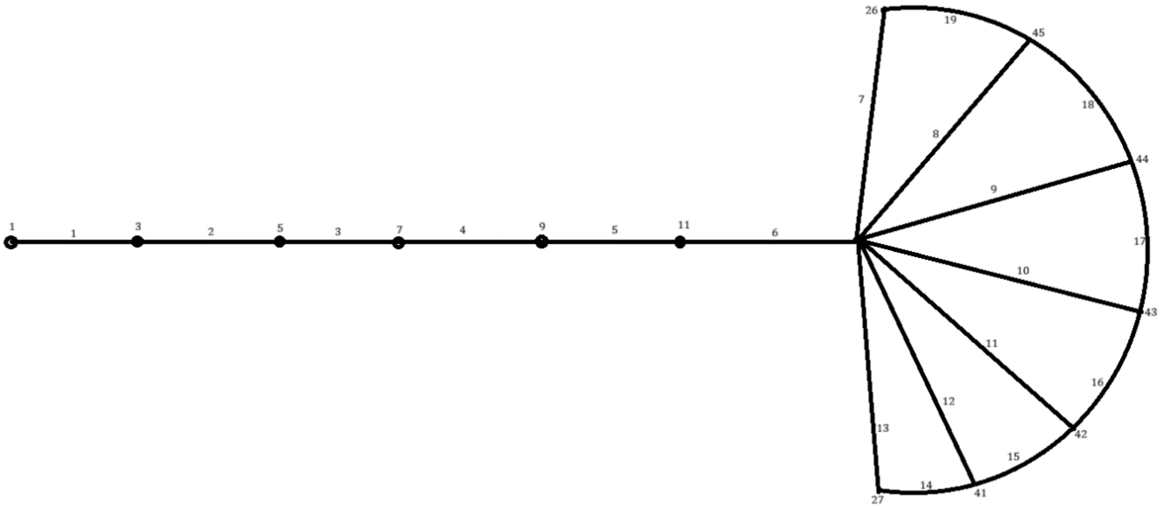

Fig 8. Vertex Antimagic Edge Slither Labeling in $U_{7,7}$.

Conclusion:

In this paper, a manipulation of path graph is considered and the admittance of vertex antimagic edge slither labeling has been established. In a similar way, different graphs can be considered and the applicability of the vertex antimagic edge slither labeling can be verified.

REFERENCES:

1. R. Bodendiek and G. Walther – On arithmetic antimagic edge labelling of graphs, Mitt. Math.Ges. Hamburg 17 (1998), 85 – 89
2. Joseph. A. Gallian – A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics (2018), ≠DS6
3. N. Hartsfield and G. Ringel – Pearls in Graph Theory, Academic Press, Boston – San Diego – New York – London, 1990.
4. Lavanya. S and Ganesan. V – Vertex Prime Labeling of Umbrella U(m,n) Graphs - International Journal of Innovative Research in Science, Engineering and Technology, Volume 9, Issue 11, November 2020.
5. Martin Baca and Mirka Miller – Super Edge Antimagic Graphs – A Wealth of Problems and Some Solutions
6. Brown Walker Press, June 2008 ISBN – 10: 1-59942-466-5 (e-book) ISBN – 13: 978-1-59942-466-8 (e-book)
7. J.A. Macdougall, M. Miller, Slamin and W.D. Wallis – Vertex Magic Total Labelling of Graphs – Utilitas Math. 61 (2002), 3 – 21
8. J.A. Macdougall, M. Miller and W.D. Wallis – Vertex Magic Total Labelling of Wheels and Related Graphs Utilitas Math. 62 (2002), 175 – 18
9. P. Mythili and S. Gokilamani – Total Coloring of Comb Related Graphs and Umbrella Graph –
10. International Journal of Creative Research Thoughts, Volume 10, Issue 5, May 2022, pp h166 – h177.
11. Sedlacek – On Magic Graphs– Math. Slovaca 26 (1976), 329 – 335.